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Linear stability analysis for opposing mixed convection in a vertical pipe

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1. INTRODUCTION

A RECENT linear stability analysis [1] for an upward flow in a heated straight pipe (aiding flow) demonstrates that the most stable fully-developed flow mode is two counter-rotating spirals. This analytical conclusion agrees with the previous experimental observations [2]. This suggests that a simple-minded parallel, fully-developed flow can only exist in an unheated pipe. In this short note the results for an opposing pipe flow are reported. This condition can be achieved for a downward flow in a heated pipe, or for an upward flow in a cooled pipe.

A linear stability analysis cannot determine whether the flow is supercritically, or subcritically, unstable. However, the experimental evidence clearly indicates that an aiding pipe flow is supercritical, and an opposing flow is subcritical. This means that, in an opposing pipe flow, the flow transition to turbulence occurs abruptly. For an aiding pipe flow, the transition process gradually passes through several equilibrium states.

A common misconception in opposing pipe flows is that buoyancy can trigger flow separation. Furthermore, parallel counter-current flows are frequently assumed in the modeling of fully-developed, non-isothermal pipe flows. It is well known that counter-current parallel flows are inviscidly unstable (the Kelvin–Helmholtz instability). The rigorous analysis reported in this note clearly shows that the linear-stability boundary for a viscous flow rules out the possible existence of parallel counter-current flows in a heated pipe. It is believed that a similar stability analysis for a developing pipe flow can demonstrate that flow transition occurs long before separation. This is one reason why neither of the flow patterns described above have been observed experimentally.

2. ANALYSIS

The analysis for an opposing flow, which is identical to that for an aiding flow, can be found in ref. [1] and is not repeated here. The only difference is that the value of the Rayleigh number Ra for an opposing flow is negative. In the following analysis, $K = -Ra$ is used.

In terms of K, the mean axial velocity is

$$W = fJ_0(K^{1/4}r) + gI_0(K^{1/4}r) \quad (1)$$

where J_n and I_n are the usual modified Bessel functions, and

$$f = \frac{K^{1/4}}{2}\,\frac{I_0(K^{1/4})}{I_0(K^{1/4})J_1(K^{1/4}) - J_0(K^{1/4})I_1(K^{1/4})}$$

$$g = -f\frac{J_0(K^{1/4})}{I_0(K^{1/4})}.$$

The associated temperature is

$$K^{1/2}\theta = f[J_0(K^{1/4}r) - J_0(K^{1/4})] - g[I_0(K^{1/4}r) - I_0(K^{1/4})]. \quad (2)$$

The mean-flow velocity profiles for $K = 0$, 50 and 100 are plotted in Fig. 1. Near the pipe wall, the fluid flows slower due to the opposing buoyancy effect. Consequently, the fluid near the center of the pipe speeds up in order to satisfy mass conservation. The inflection point of the velocity profile indicates that the flow is likely to be unstable. The corresponding distributions of mean temperature are given in Fig. 2.

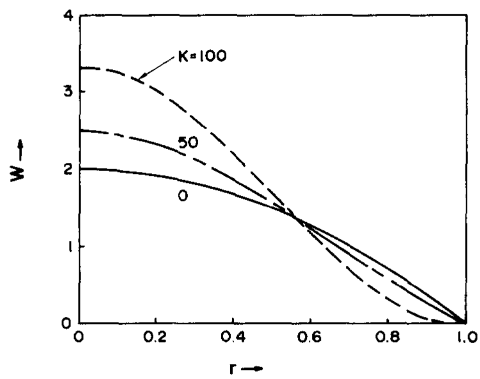

FIG. 1. Mean-flow velocity profiles.

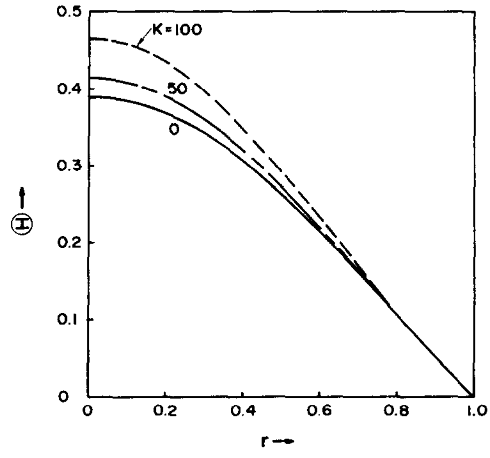

FIG. 2. Mean temperature distribution.

Due to the different nondimensionalization of the equations in refs. [1, 2], the relation between the governing parameters is needed to compare results: $(Gr/Re)_2$ in ref. [2] equals $K\cdot\theta(0)$ in ref. [1]. The centerline temperature $\theta(0)$ as a function of K and the relation between $(Gr/Re)_2$ and K are provided in Fig. 3.

Substitution of equations (1) and (2) into the linear stability equations in ref. [1] results in the eigenvalue problem which describes the stability boundary for opposing flows. The solution of this problem proceeds in the manner discussed in ref. [1].

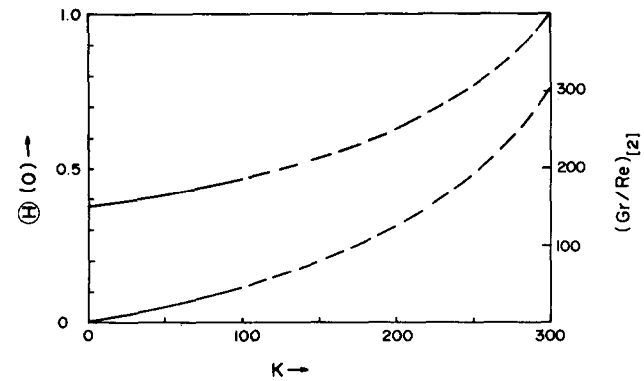

FIG. 3. Centerline temperature and relation of Gr/Re in ref. [2] with K.

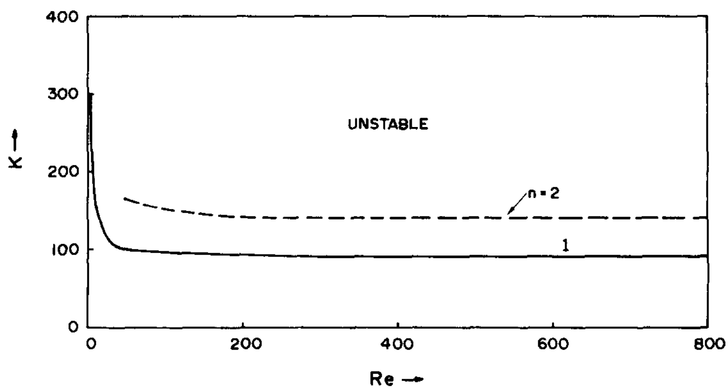

FIG. 4. Flow instability boundary in (Re, K) planes.

3. RESULTS AND DISCUSSION

The numerical results indicate that the flow is unconditionally stable for an axisymmetric disturbance, $n = 0$. The flow instability boundaries for $n = 1$ and 2 are plotted in Fig. 4. It is clear that $n = 1$ is the most unstable mode. The associated eigenmode is two counter-rotating spirals. Since the experimental observation shows that the flow is subcritical and the flow transition is rather abrupt, it is difficult to detect this flow pattern in the laboratory. On the other hand, the experimental observations in ref. [2] show a tendency for the velocity profiles to become asymmetric before an unsteady motion sets in. For $Re > 50$, the analysis shows that the flow is unstable for $K > 100$. This rules out the existence of parallel counter-current flows as the fully-developed state in a heated pipe. Scheele and Hanratty [2] have speculated on the existence of this phenomenon, which is now analytically confirmed. For $Re < 50$, the results are not reliable. This is because the velocities are scaled by the averaged axial velocity which is zero at $Re = 0$. It is worthwhile to note that the fluid is unstably stratified for opposing pipe flows. Without forced flow, the fluid layer is unstable (the well-known Taylor instability).

In conclusion, the linear-stability boundary for opposing flows convincingly demonstrates that a simple-minded parallel counter-current flow cannot exist as a fully-developed flow in a heated vertical pipe due to flow instability.

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